LECTURE NOTE

ON

Engineering Mechanics

DIPLOMA 1st Year

DEPARTMENT OF MECHANICAL ENGINEERING

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(Approved by AICTE)

ANDHARUA,BHUBANESWAR

Objective:

On completion of the subject, the student will be able to do:

1. Compute the force, moment & their application through solving of simple problems on coplanar forces.
2. Understand the concept of equilibrium of rigid bodies.
3. Know the existence of friction & its applications through solution of problems on above.
4. Locate the C.G. & find M.I. of different geometrical figures.
5. Know the application of simple lifting machines.
6. Understand the principles of dynamics.

1. FUNDAMENTALS OF ENGINEERING MECHANICS

1.1 Fundamentals.

Definitions of Mechanics, Statics, Dynamics, Rigid Bodies,

1.2 Force

Force System.

Definition, Classification of force system according to plane & line of action. Characteristics of Force & effect of Force. Principles of Transmissibility & Principles of Superposition. Action & Reaction Forces & concept of Free Body Diagram.

1.3 Resolution of a Force.

Definition, Method of Resolution, Types of Component forces, Perpendicular components & non-perpendicular components.

1.4 Composition of Forces.

Definition, Resultant Force, Method of composition of forces, such as

1.4.1 Analytical Method such as Law of Parallelogram of forces & method of resolution.

1.4.2. Graphical Method.

Introduction, Space diagram, Vector diagram, Polygon law of forces.

1.4.3 Resultant of concurrent, non-concurrent & parallel force system by Analytical & Graphical Method.

1.5 Moment of Force.

Definition, Geometrical meaning of moment of a force, measurement of moment of a force & its S.I units. Classification of moments according to direction of rotation, sign convention, Law of moments, Varignon's Theorem, Couple – Definition, S.I. units, measurement of couple, properties of couple.

2. EQUILIBRIUM

2.1 Definition, condition of equilibrium, Analytical & Graphical conditions of equilibrium for concurrent, non-concurrent & Free Body Diagram.

2.2 Lamia's Theorem – Statement, Application for solving various engineering problems.

3. FRICTION

3.1 Definition of friction, Frictional forces, Limiting frictional force, Coefficient of Friction.

Angle of Friction & Repose, Laws of Friction, Advantages & Disadvantages of Friction.

3.2 Equilibrium of bodies on level plane – Force applied on horizontal & inclined plane (up &down).

3.3 Ladder, Wedge Friction.

4. CENTROID & MOMENT OF INERTIA

4.1 Centroid – Definition, Moment of an area about an axis, centroid of geometrical figures such as squares, rectangles, triangles, circles, semicircles & quarter circles, centroid of composite figures.

4.2 Moment of Inertia – Definition, Parallel axis & Perpendicular axis Theorems. M.I. of plane lamina & different engineering sections.

5. SIMPLE MACHINES

5.1 Definition of simple machine, velocity ratio of simple and compound gear train, explain simple & compound lifting machine, define M.A, V.R. & Efficiency & State the relation between them, State Law of Machine, Reversibility of Machine, Self Locking Machine.

5.2 Study of simple machines – simple axle & wheel, single purchase crab winch & double purchase crab winch, Worm & Worm Wheel, Screw Jack.
5.3 Types of hoisting machine like derricks etc, Their use and working principle. No problems.

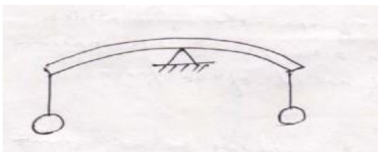
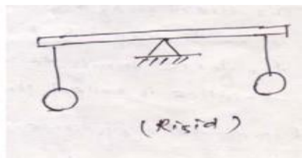
6. DYNAMICS

6.1 Kinematics & Kinetics, Principles of Dynamics, Newton’s Laws of Motion, Motion of Particle acted upon by a constant force, Equations of motion, De-Alembert’s Principle.
6.2 Work, Power, Energy & its Engineering Applications, Kinetic & Potential energy & its application.
6.3 Momentum & impulse, conservation of energy & linear momentum, collision of elastic bodies, and Coefficient of Restitution.

Mechanics:- It is defined as that branch of science, which describes and predicts the conditions of rest or motion of bodies under the action of forces. Engineering mechanics applies the principle of mechanics to design, taking into account the effects of forces.

Statics :-Statics deal with the condition of equilibrium of bodies acted upon by forces.

Rigid body:- A rigid body is defined as a definite quantity of matter, the parts of which are fixed in position relative to each other. Physical bodies are never absolutely but deform slightly under the action of loads. If the deformation is negligible as compared to its size, the body is termed as rigid.

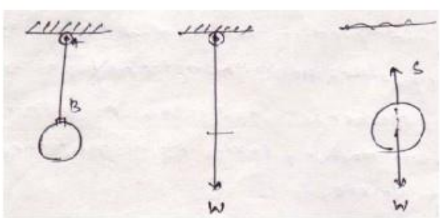



Dynamics

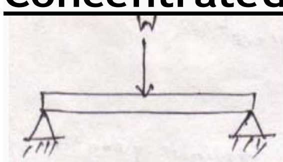
It deals with study of motion and forces on a body which is in motion. Subcomponent of dynamics is kinematics.

Force :-Force may be defined as any action that tends to change the state of rest or motion of a body to which it is applied. The three quantities required to completely define force are called its specification or characteristics. So the characteristics of a force are:

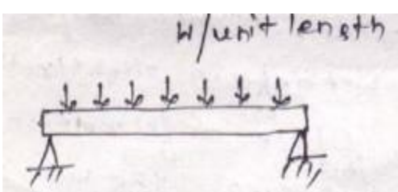
1. Magnitude 2. Point of application 3. Direction of application



Concentrated force/point load-

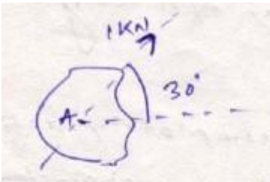


Distributed force



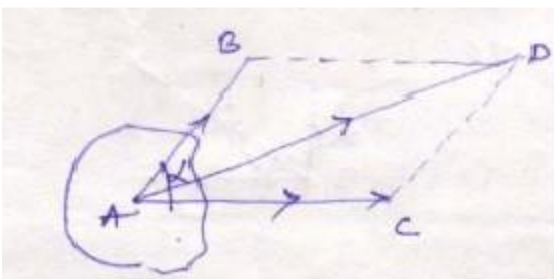
Line of action of force:- The direction of a force is the direction, along a straight line through its point of application in which the force tends to move a body when it is applied. This line is called line of action of force.

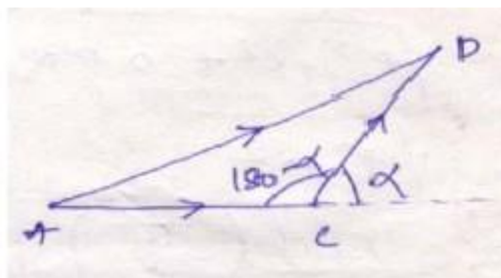
Representation of force:- Graphically a force may be represented by the segment of a straight line.



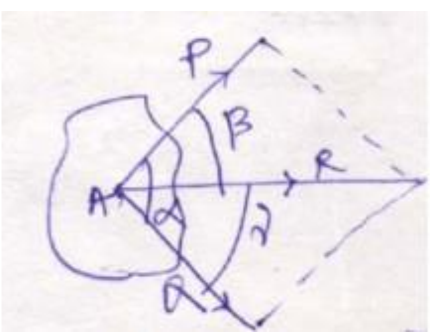
Composition of two forces :-The reduction of a given system of forces to the simplest system that will be its equivalent is called the problem of composition of forces.

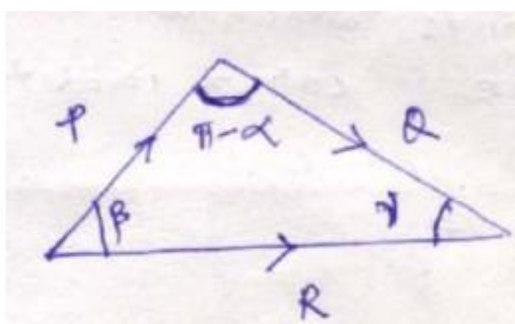
Parallelogram law :-If two forces represented by vectors AB and AC acting under an angle α are applied to a body at point A. Their action is equivalent to the action of one force, represented by vector AD, obtained as the diagonal of the parallelogram constructed on the vectors AB and AC directed as shown in the figure.





Force AD is called the resultant of AB and AC and the forces are called its components.





$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos\alpha\right)}$$

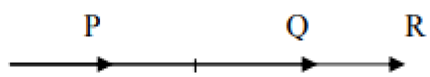
Now applying triangle law

$$\frac{P}{Sin\gamma} = \frac{Q}{Sin\beta} = \frac{R}{Sin(\pi - \alpha)}$$

Special cases

Case-I: If $\alpha = 0^\circ$

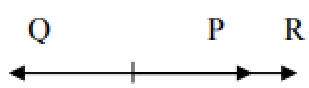
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos0^\circ\right)} = \sqrt{(P+Q)^2} = P + Q$$



R = P+Q

Case- II: If $\alpha = 180^\circ$

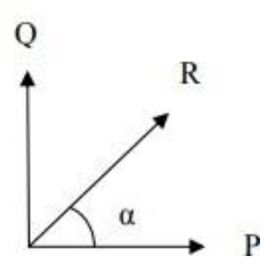
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos180^\circ\right)} = \sqrt{(P^2 + Q^2 - 2PQ)} = \sqrt{(P-Q)^2} = P - Q$$



Case-III: If $\alpha = 90^\circ$

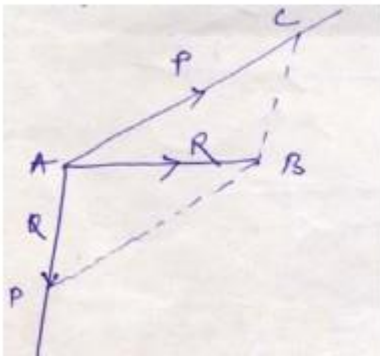
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos90^\circ\right)} = \sqrt{P^2 + Q^2}$$

$\alpha = \tan^{-1}(Q/P)$



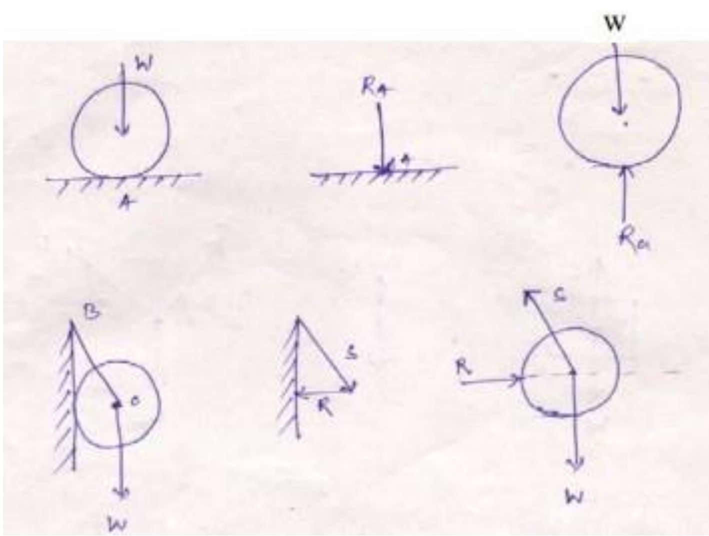
Resolution of a force

The replacement of a single force by a several components which will be equivalent in action to the given force is called resolution of a force.



Action and reaction

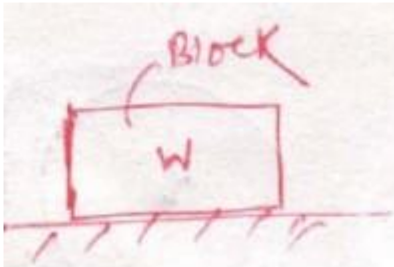
Often bodies in equilibrium are constrained to investigate the conditions.

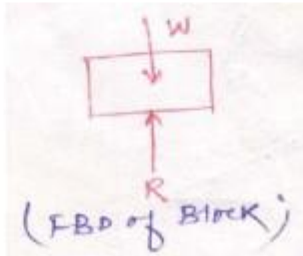


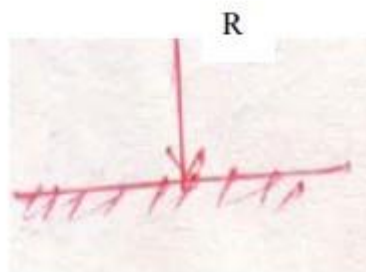
Free body diagram

Free body diagram is necessary to investigate the condition of equilibrium of a body or system. While drawing the free body diagram all the supports of the body are removed and replaced with the reaction forces acting on it.

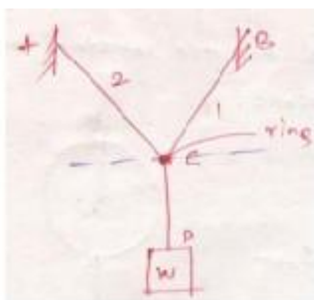
1. Draw the free body diagrams of the following figures.

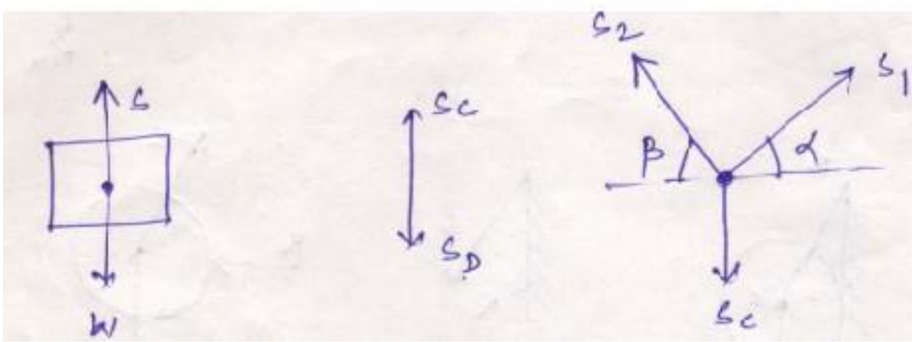




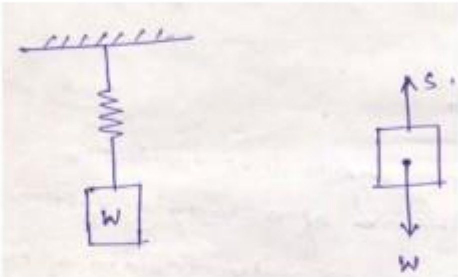


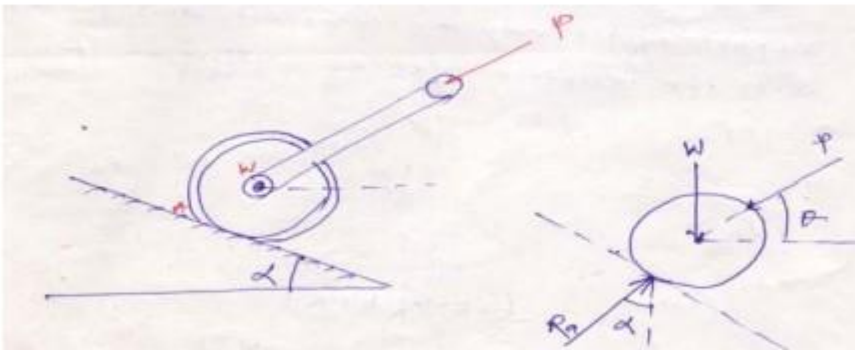
2. Draw the free body diagram of the body, the string CD and the ring.





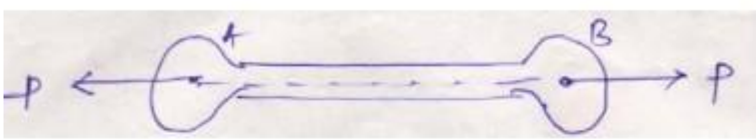
3. Draw the free body diagram of the following figures.



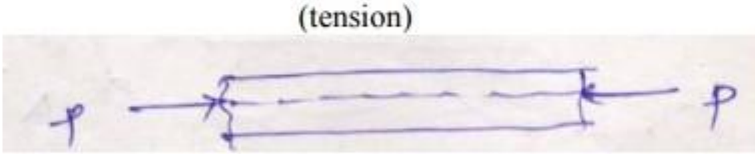


<u>**Equilibrium of colinear forces:**</u>

Equllibrium law: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.



(tension)



(compression)

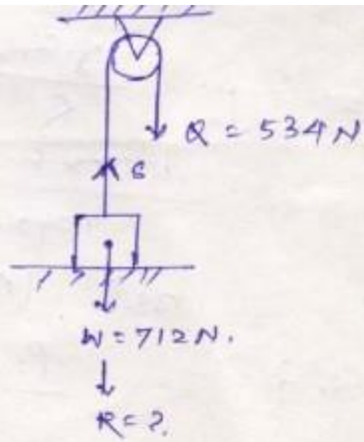
<u>**Superposition and transmissibility**</u>

Problem 1: A man of weight W = 712 N holds one end of a rope that passes over a pulley vertically above his head and to the other end of which is attached a weight Q = 534 N. Find the force with which the man's feet press against the floor.

Tension in the string S is equal to the load attached to it

Q = 534 N.

So $\boxed{S = 534\ N.}$



Now applying parallelogram law resultant force

$R = \sqrt{W^2 + S^2 + 2WS\cos 180^\circ}$

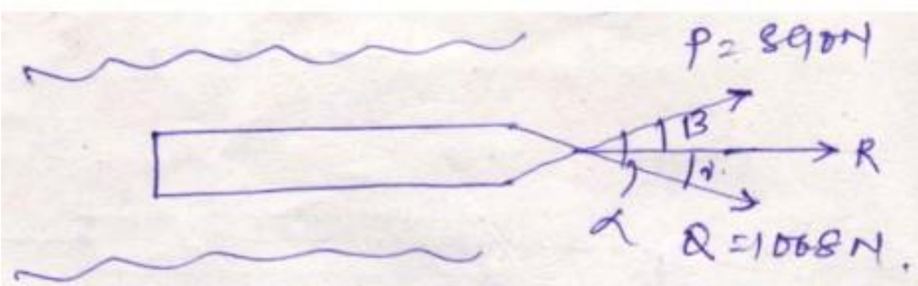
$= \sqrt{W^2 + S^2 - 2WS}$

$= \sqrt{(W-S)^2} = W - S$

$\Rightarrow R = 712 - 534 = 178N\ (\downarrow)$

Reaction on the man's feet $= 178N\ (\uparrow)$

Problem 2: A boat is moved uniformly along a canal by two horses pulling with forces P = 890 N and Q = 1068 N acting under an angle α = 60˚. Determine the magnitude of the resultant pull on the boat and the angles β and ν.



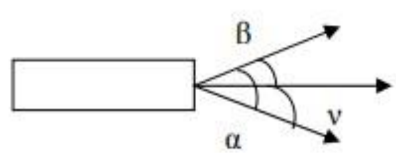
P = 890 N, α = 60˚

Q = 1068 N

$R = \sqrt{(P^2 + Q^2 + 2PQ\cos\alpha)}$

$= \sqrt{(890^2 + 1068^2 + 2 \times 890 \times 1068 \times 0.5)}$

$= 1698.01N$

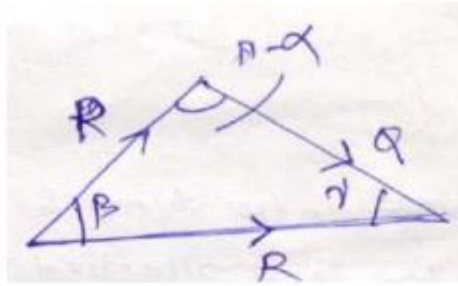


$$\frac{Q}{\sin\beta} = \frac{P}{\sin\nu} = \frac{R}{\sin(\pi-\alpha)}$$

$$\sin\beta = \frac{Q\sin\alpha}{R}$$

$$= \frac{1068 \times \sin 60^{\circ}}{1698.01}$$

$$= 33^{\circ}$$



$$\sin\nu = \frac{P\sin\alpha}{R}$$

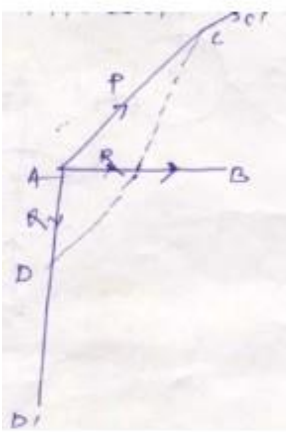
$$= \frac{890 \times \sin 60^{\circ}}{1698.01}$$

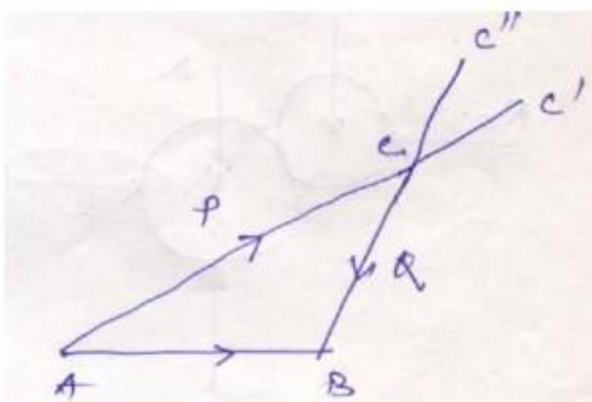
$$= 27^{\circ}$$

Resolution of a force

Replacement of a single force by several components which will be equivalent in action to the given force is called the problem of resolution of a force.

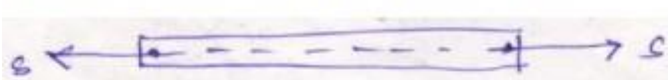
By using parallelogram law, a single force R can be resolved into two components P and Q intersecting at a point on its line of action.





Equilibrium of collinear forces:

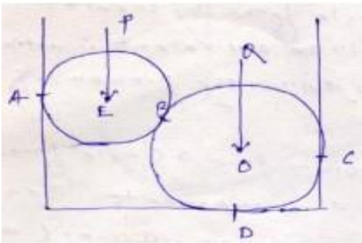
Equilibrium law: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.

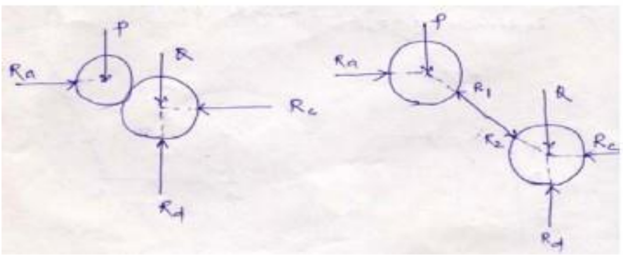


Law of superposition

The action of a given system of forces on a rigid body will no way be changed if we add to or subtract from them another system of forces in equllibrium.

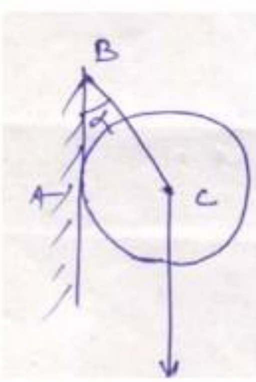
Problem 3: Two spheres of weight P and Q rest inside a hollow cylinder which is resting on a horizontal force. Draw the free body diagram of both the spheres, together and separately.





Equilibrium of concurrent forces in a plane

- If a body known to be in equilibrium is acted upon by several concurrent, coplanar forces, then these forces or rather their free vectors, when geometrically added must form a closed polygon.
- This system represents the condition of equilibrium for any system of concurrent forces in a plane.



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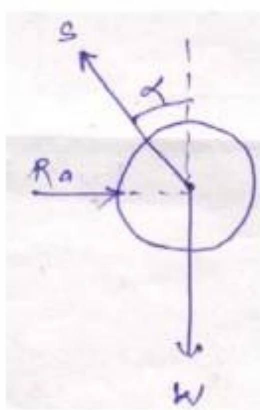
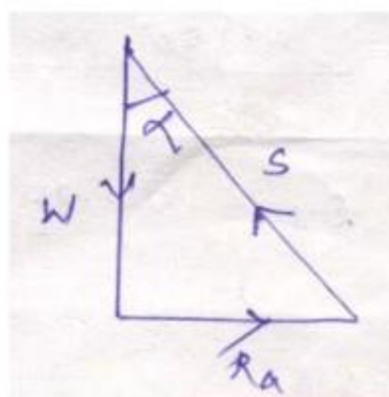
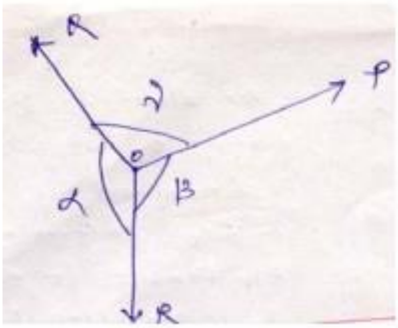




Lami's theorem

If three concurrent forces are acting on a body kept in an equllibrium, then each force is proportional to the sine of angle between the other two forces and the constant of proportionality is same.

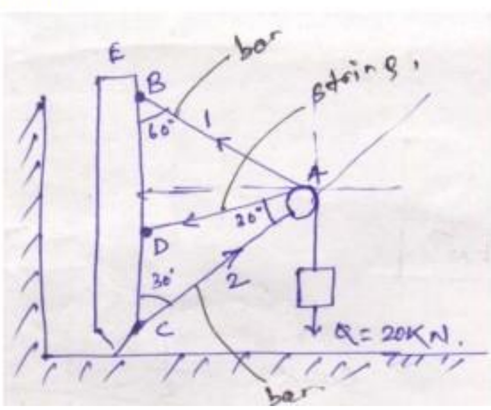


$$\frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\upsilon}$$

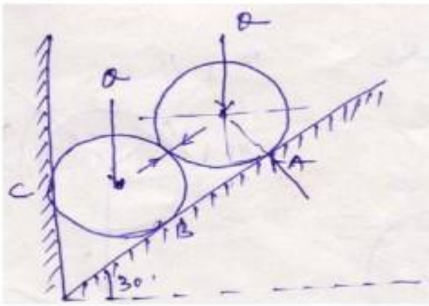
<u>Theory of transmissiibility of a force:</u>

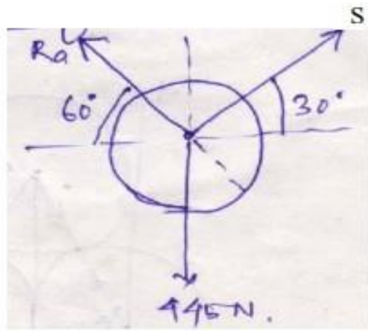
The point of application of a force may be transmitted along its line of action without changing the effect of force on any rigid body to which it may be applied.

Problem:

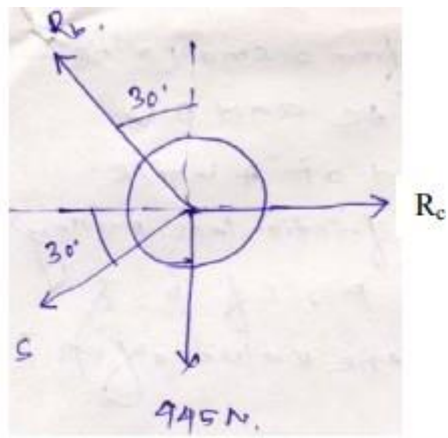


Problem: Two identical rollers each of weight Q = 445 N are supported by an inclined plane and a vertical wall as shown in the figure. Assuming smooth surfaces, find the reactions induced at the points of support A, B and C.





$$\frac{R_a}{\sin 120} = \frac{S}{\sin 150} = \frac{445}{\sin 90}$$



Resolving vertically

$$\sum Y = 0$$

$$R_b \cos 60 = 445 + S \sin 30$$

$$\Rightarrow R_b \frac{\sqrt{3}}{2} = 445 + \frac{222.5}{2}$$

$$\Rightarrow R_b = 642.302 N$$

Resolving horizontally

$$\sum X = 0$$

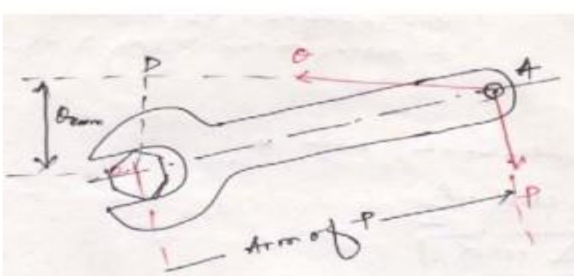
$$R_c = R_b \sin 30 + S \cos 30$$

$$\Rightarrow 642.302 \sin 30 + 222.5 \cos 30$$

$$\Rightarrow R_c = 513.84 N$$

Method of moments

Moment of a force with respect to a point:



- Considering wrench subjected to two forces P and Q of equal magnitude. It is evident that force P will be more effective compared to Q, though they are of equal magnitude.
- The effectiveness of the force as regards it is the tendency to produce rotation of a body about a fixed point is called the moment of the force with respect to that point.
- Moment = Magnitude of the force × Perpendicular distance of the line of action of force.
- Point O is called moment centre and the perpendicular distance (i.e. OD) is called moment arm.
- Unit is N.m

Theorem of Varignon:

The moment of the resultant of two concurrent forces with respect to a centre in their plane is equal to the alzebric sum of the moments of the components with respect to some centre.

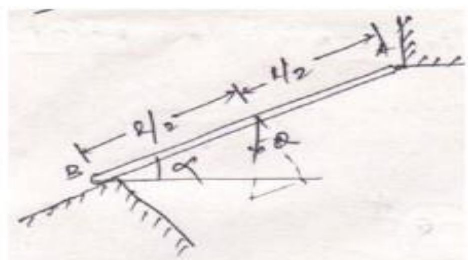
Problem 1:

A prismatic clear of AB of length l is hinged at A and supported at B. Neglecting friction, determine the reaction R_b produced at B owing to the weight Q of the bar.

Taking moment about point A,

$$R_b \times l = Q\cos\alpha.\frac{l}{2}$$

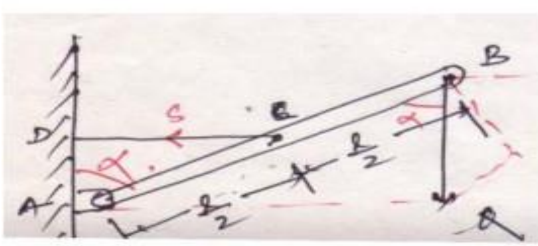
$$\Rightarrow R_b = \frac{Q}{2}\cos\alpha$$



P

roblem 2:

A rigid bar AB is supported in a vertical plane and carrying a load Q at its free end. Neglecting the weight of bar, find the magnitude of tensile force S in the horizontal string CD.



Taking moment about A,

$$\sum M_A = 0$$

$$S.\frac{l}{2}\cos\alpha = Q.l\sin\alpha$$

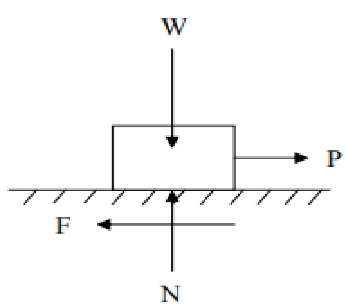
$$\Rightarrow S = \frac{Q.l\sin\alpha}{\frac{l}{2}\cos\alpha}$$

$$\Rightarrow S = 2Q.\tan\alpha$$

Friction

- The force which opposes the movement or the tendency of movement is called **Frictional force or simply friction**. It is due to the resistance to motion offered by minutely projecting particles at the contact surfaces. However, there is a limit beyond which the magnitude of this force cannot increase.
- If the applied force is more than this limit, there will be movement of one body over the other. This limiting value of frictional force when the motion is impending, it is known as **Limiting Friction**.
- When the applied force is less than the limiting friction, the body remains at rest and such frictional force is called **Static Friction**, which will be having any value between zero and the limiting friction.
- If the value of applied force exceeds the limiting friction, the body starts moving over the other body and the frictional resistance experienced by the body while moving is known as **Dynamic Friction**. Dynamic friction is less than limiting friction.
- Dynamic friction is classified into following two types:
    a) Sliding friction
    b) Rolling friction

- Sliding friction is the friction experienced by a body when it slides over the other body.
- Rolling friction is the friction experienced by a body when it rolls over a surface.
- It is experimentally found that the magnitude of limiting friction bears a constant ratio to the normal reaction between two surfaces and this ratio is called **Coefficient of Friction**.



$$\text{Coefficient of friction} = \frac{F}{N}$$

where F is limiting friction and N is normal reaction between the contact surfaces.

Coefficient of friction is denoted by μ.

Thus, $\mu = \frac{F}{N}$

Laws of friction

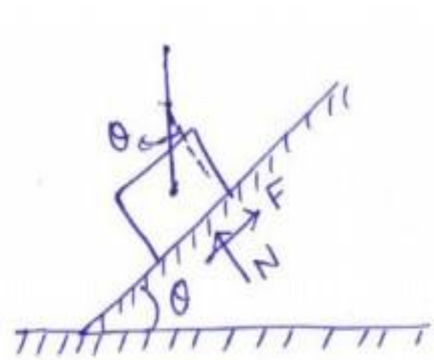
1. The force of friction always acts in a direction opposite to that in which body tends to move.
2. Till the limiting value is reached, the magnitude of friction is exactly equal to the force which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces of contact and this ratio is called coefficient of friction.
4. The force of friction depends upon the roughness/smoothness of the surfaces.
5. The force of friction is independent of the area of contact between the two surfaces.
6. After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio with normal force. This ratio is called **coefficient of dynamic friction**.

Angle of friction

Consider the block shown in figure resting on a horizontal surface and subjected to horizontal pull P. Let F be the frictional force developed and N the normal reaction. Thus, at contact surface the reactions are F and N. They can be graphically combined to get the reaction R which acts at angle θ to normal reaction. This angle θ called the angle of friction is given by

This value of α is called Angle of Limiting Friction. Hence, the angle of limiting friction may be defined as the angle between the resultant reaction and the normal to the plane on which the motion of the body is impending.

Angle of repose



Consider the block of weight W resting on an inclined plane which makes an angle θ with the horizontal. When θ is small, the block will rest on the plane. If θ is gradually increased, a stage is reached at which the block start sliding down the plane. The angle θ for which the motion is impending, is called the angle of repose. Thus, the maximum inclination of the plane on which a body, free from external forces, can repose is called **Angle of Repose**.

Resolving vertically,
$N = W.\cos\theta$

Resolving horizontally,
$F = W.\sin\theta$

Thus, $\tan\theta = \frac{F}{N}$

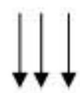
If ϕ is the value of θ when the motion is impending, the frictional force will be limiting friction and hence,

$$\tan\phi = \frac{F}{N}$$
$$= \mu = \tan\alpha$$
$$\Rightarrow \phi = \alpha$$

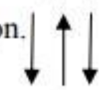
Thus, the value of angle of repose is same as the value of limiting angle of repose.

Parallel forces on a plane

Like parallel forces: Coplanar parallel forces when act in the same direction.

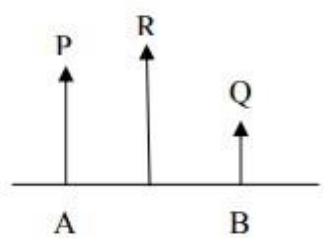


Unlike parallel forces: Coplanar parallel forces when act in different direction.



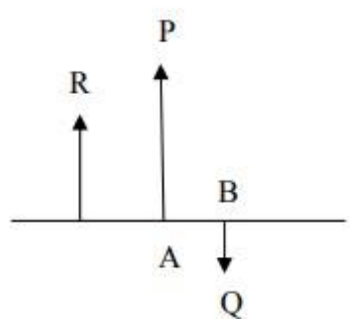
Resultant of like parallel forces:

Let P and Q are two like parallel forces act at points A and B.
R = P + Q



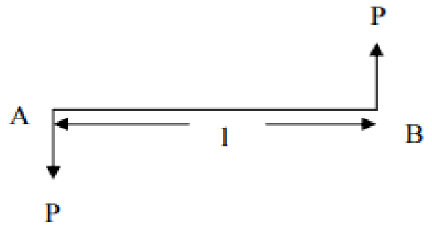
Resultant of unlike parallel forces:
R = P - Q

R is in the direction of the force having greater magnitude.



Couple:

Two unlike equal parallel forces form a couple.

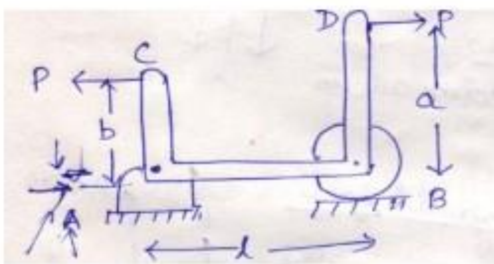


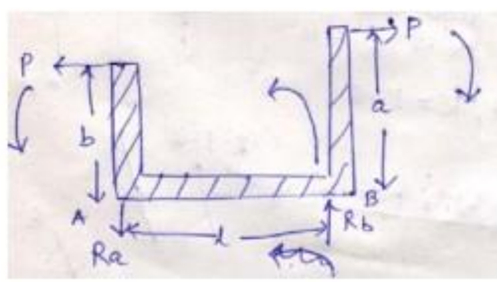
The rotational effect of a couple is measured by its moment.

Moment = P × l

Sign convention: Anticlockwise couple (Positive)
Clockwise couple (Negative)

Problem 1 : A rigid bar CABD supported as shown in figure is acted upon by two equal horizontal forces P applied at C and D. Calculate the reactions that will be induced at the points of support. Assume l = 1.2 m, a = 0.9 m, b =0.6 m.





$$\sum V = 0$$

$$R_a = R_b$$

Taking moment about A,

$$R_a = R_b$$

$$R_b \times l + P \times b = P \times a$$

$$\Rightarrow R_b = \frac{P(0.9 - 0.6)}{1.2}$$

$$\Rightarrow R_b = 0.25P(\uparrow)$$

$$\Rightarrow R_a = 0.25P(\downarrow)$$

Centre of gravity

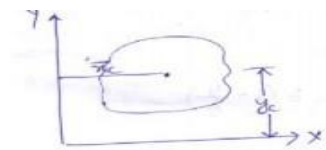
Centre of gravity: It is that point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space.

- As the point through which resultant of force of gravity (weight) of the body acts.

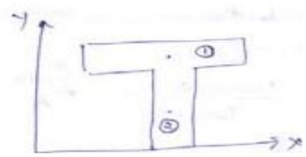
Centroid: Centrroid of an area lies on the axis of symmetry if it exits.

Centre of gravity is applied to bodies with mass and weight and centroid is applied to plane areas.

$$x_c = \sum A_i x_i$$
$$y_c = \sum A_i y_i$$

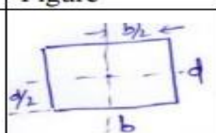
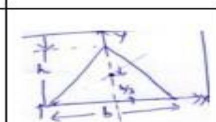
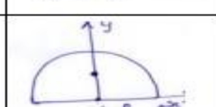
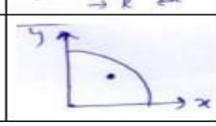


$$x_c = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$
$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

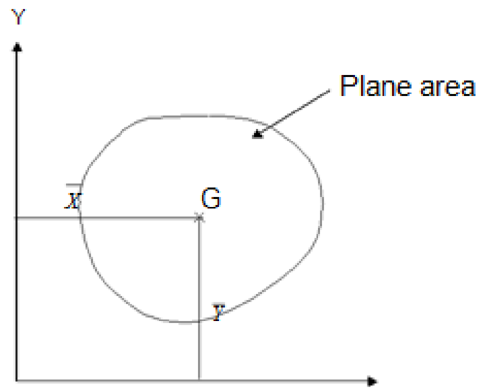


$$x_c = y_c = \frac{\text{Moment of area}}{\text{Total area}}$$
$$x_c = \frac{\int x.dA}{A}$$
$$y_c = \frac{\int y.dA}{A}$$

Centroids of different figures

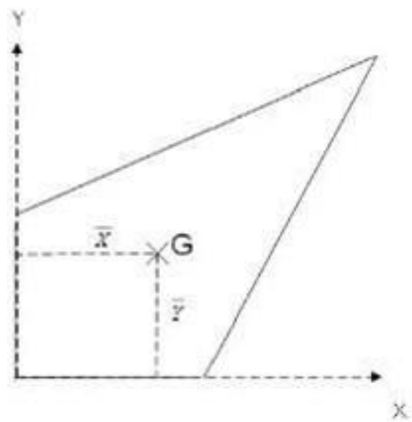
Shape	Figure	$\overline{x}$	$\overline{y}$	Area
Rectangle		$\frac{b}{2}$	$\frac{d}{2}$	bd
Triangle		0	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{4}$

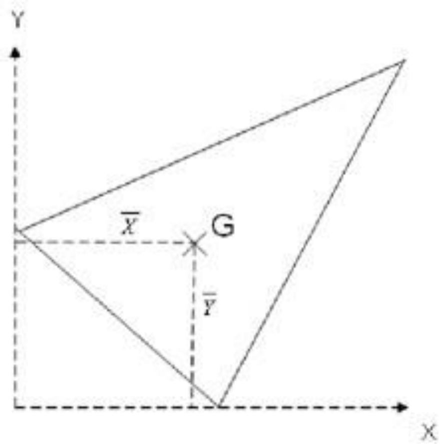
Location of centroid of plane areas



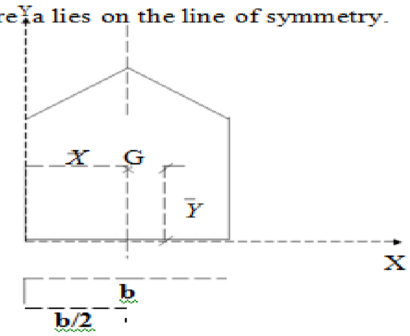
The position of centroid of a plane area should be specified or calculated with respect to some reference axis i.e. X and Y axis. The distance of centroid G from vertical reference axis or Y axis is denoted as X and the distance of centroid G from a horizontal reference axis or X axis is denoted as Y.

While locating the centroid of plane areas, a bottommost horizontal line or a horizontal line through the bottommost point can be made as the X – axis and a leftmost vertical line or a vertical line passing through the leftmost point can be made as Y- axis.





In some cases the given figure is symmetrical about a horizontal or vertical line such that the centroid of the plane area lies on the line of symmetry.



The above figure is symmetrical about a vertical line such that G lies on the line of symmetry. Thus

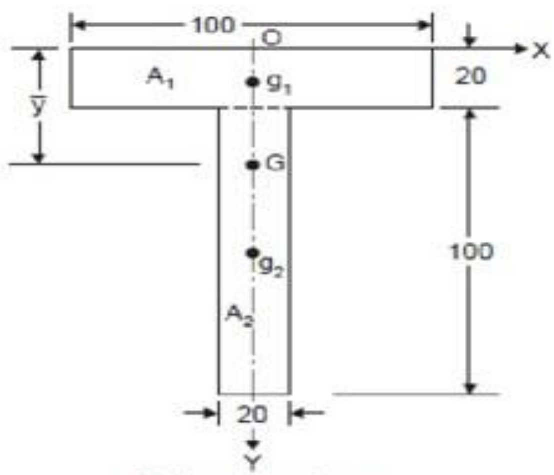
X= b/2.

Y=?

The centroid of plane geometric area can be located by one of the following methods

a) Graphical methods
b) Geometric consideration
c) Method of moments

Q) Locate the centroid of the T-section shown in fig.



All dimensions in mm

Solution. Selecting the axis as shown in Fig. we can say due to symmetry centroid lies on y axis, *i.e.* $\bar{x} = 0$. Now the given T-section may be divided into two rectangles A_1 and A_2 each of size 100 × 20 and 20 × 100. The centroid of A_1 and A_2 are $g_1(0, 10)$ and $g_2(0, 70)$ respectively.

∴ The distance of centroid from top is given by:

$$\bar{y} = \frac{100 \times 20 \times 10 + 20 \times 100 \times 70}{100 \times 20 + 20 \times 100}$$

$$= 40 \text{ mm}$$

Hence, *centroid of T-section is on the symmetric axis at a distance 40 mm from the top.* **Ans.**

RADIUS OF GYRATION k

The **radius of gyration** of an area with respect to a particular axis is the square root of the quotient of the moment of inertia divided by the area. It is the distance at which the entire area must be assumed to be concentrated in order that the product of the area and the square of this distance will equal the moment of inertia of the actual area about the given axis. In other words, the radius of gyration describes the way in which the total cross-sectional area is distributed around its centroidal axis. If more area is distributed further from the axis, it will have greater resistance to buckling. The most efficient column section to resist buckling is a circular pipe, because it has its area distributed as far away as possible from the centroid.

Rearranging we have:

$$I_x = k_x^2 A$$
$$I_y = k_y^2 A$$

The radius of gyration is the distance *k* away from the axis that all the area can be concentrated to result in the same moment of inertia.

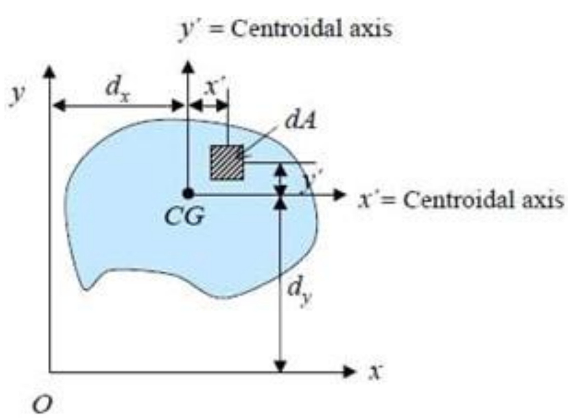
$$k_x = \sqrt{\frac{I_x}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

Parallel Axis Theorem

The moment of inertia of an area with respect to any given axis is equal to the moment of inertia with respect to the centroidal axis plus the product of the area and the square of the distance between the 2 axes.

The parallel axis theorem is used to determine the moment of inertia of composite sections.



$$I_x = \int_A (y'+d_y)^2\, dA$$

$$= \int_A [(y')^2 + 2(y')(d_y) + (d_y)^2]\, dA$$

$$= \int_A (y')^2\, dA + \int_A 2(y')(d_y)\, dA + \int_A (d_y)^2\, dA$$

$0,\ \bar{y}' = 0$

$$= \bar{I}_x + 2d_y \int_A y'\, dA + d_y^2 \int_A dA$$

$$I_x = \bar{I}_x + 0 + d_y^2 A$$

$$I_y = \bar{I}_y + 0 + d_x^2 A$$

$$J_O = \bar{J}_C + Ad^2$$

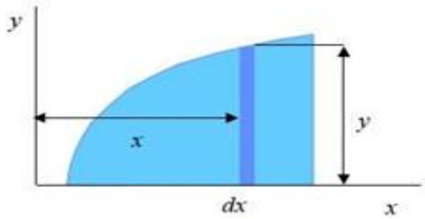
Perpendicular Axis Theorem

Theorem of the perpendicular axis states that if I_{XX} and I_{YY} be the moment of inertia of a plane section about two mutually perpendicular axis X-X and Y-Y in the plane of the section, then the moment of inertia of the section I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$I_{ZZ} = I_{XX} + I_{YY}$$

The moment of inertia I_{ZZ} is also known as polar moment of inertia.

Determination of the moment of inertia of an area by integration



The ***rectangular moments of inertia*** I_x ***and*** I_y ***of an area*** are defined as

$$I_x = \int y^2 \, dA \qquad I_y = \int x^2 \, dA$$

Simple Machine or Lifting Machine: A machine a device by which heavy load can be lifted by applying less effort as compared to the load.e.g. Heavy load of car can be lifted with the help of simple screw jack by applying small force.

Compound Machine:

Compound machine is a device which may consists of number of simple machines. A compound machine may also be defined as a machine which has multiple mechanisms for the same purpose.
Compound machines do heavy work with less efforts and greater speed.

e.g. In a crane, one mechanism (gears) are used to drive the rope drum and other mechanism (pulleys) are used to lift the load. Thus, a crane consists of two simple machines or mechanisms i.e. gears and pulleys. Hence, it is a compound machine.

Effort:

It may be defined as, the force which is applied so as to overcome the resistance or to lift the load.It is denoted by „P‟.Magnitude of effort (P) is small as compared to the load (W).

Load:

The weight to be lifted or the resistive force to be overcome with the help of a machine is called as load (W).

Velocity Ratio (V.R.):

It is defined as the ration of distance traveled by the effort (P) to the distance traveled by the load (W)

$$V.R. = \frac{\text{Distance travelled by effort}}{\text{Distance travelled by load}}$$

Velocity ratio will be always more than one and for a given machine, it remains constant.

Mechanical Advantage:

It is defined as the ratio of load to be lifted to the effort applied

$$M.A. = \frac{\text{Load (W)}}{\text{Effort (P)}} = \frac{W}{P}$$

Input:

The amount of work done by the effort is called as input and is equal to the product of effort and distance travelled by it.

Input = P x X, where, P – Effort and X – distance travelled by the effort

Output:

The amount of work done by the load is called as output and is equal to the product of load and distance travelled by it.

Output = W x Y where, W – Load and Y – distance travelled by the load

Efficiency:

The ratio of output to input is called as efficiency of machine and it is denoted by Greek letter eta (η)

Generally, efficiency is expressed in percentage

$$\%\eta = \frac{\text{Output}}{\text{Input}} \times 100$$

It is always less than 100 because of friction, therefore output < input.

But Output = W.Y and Input = P.X

$$\%\eta = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{W \times Y}{P \times X} \times 100$$

Therefore, efficiency of a machine is also defined as the ratio of mechanical advantage (M.A.) to the velocity ratio (V.R.). It is also expressed in percentage.

$$\% \eta = \frac{M.A.}{V.R.} x100$$

It is always less than 100 because of friction, therefore M.A. < V.R.

Actual Machine:

The machine whose efficiency is always less than 100 % due to frictional resistance offered by the different moving component parts of the machine is called as actual machine.

For such machines, η < 100 % and hence M.A. < V.R.

Ideal Machine:

The machine whose efficiency is 100 % and in which friction is totally absent or zero, is called as ideal machine.

For ideal machines, η = 100 % and hence M.A. = V.R.

Ideal Effort (Pi):

The effort which is required to lift the load when there is no friction is called as an ideal effort (Pi)

$$\text{Ideal Effort Pi} = \frac{W}{V.R.}$$

Where, Pi = Ideal Effort, W = Load to be lifted, V.R. = Velocity Ratio

Ideal Load (Wi):

The load which can be lifted by an effort (P), when there is no friction, is called as an ideal load (Wi)

Ideal Load Wi = P x V.R.

Where, P = Effort applied, Wi = Ideal Load, V.R. = Velocity Ratio

Maximum Mechanical Advantage (Max. M.A.):

We know that,

$$M.A. = \frac{W}{P}$$

$$\text{But, } P = mW + C$$

$$\therefore M.A. = \frac{W}{mW + C}$$

Dividing the numerator & Denominator by ' W' we get

$$M.A. = \frac{1}{m + \frac{C}{W}}$$

In the above equation if ' W' is more, the ratio $\frac{C}{W}$ will be very small ∴ Neglectingthe ratio $\frac{C}{W}$, M.A. will be maximum

Maximum Efficiency:

The ratio of maximum M.A. to the V.R. is called as maximum efficiency.

It is also expressed in percentage as

$$\therefore \% \text{ Maximum } \eta = \frac{\text{Max M.A.}}{\text{V.R.}} \times 100 = \frac{1}{m} \times \frac{1}{\text{V.R.}} \times 100 \qquad \left(\text{Since Max M.A.} = \frac{1}{m}\right)$$

Reversible Machine:

When a machine is capable of doing some work in the reverse direction even on removal of effort, it is called as reversible machine.

Condition for Reversible Machine:

The efficiency of the machine should be more than 50%.

Irreversible Machine / Non-reversible Machine / Self Locking Machine:

When a machine is not capable of doing some work in the reverse direction even on removal of effort, it is called as irreversible machine or non-reversible machine or self locking machine.

Condition for Irreversible Machine:

The efficiency of the machine should be less than 50%.

Friction in Machines in terms of Effort and Load:

In any machine, there are number of parts which are in contact with each other in their relative motion. Hence, there is always a frictional resistance and due to which the machine is unable to produce 100 % efficiency.

Let, P = Actual Effort, P_f = Effort Lost in friction, Pi = Ideal Effort

∴ **Effort Lost in friction (P_f) = Actual Effort (P) – Ideal Effort (Pi)**

$$\therefore P_f = P - P_i = P - \frac{W}{\text{V.R.}} \qquad \left(\text{Since } P_i = \frac{W}{\text{V.R.}}\right)$$

Let, W = Actual load lifted, W_f = Load Lost in friction, Wi = Ideal Load

∴ **Load Lost in friction (W_f) = Ideal Load (Wi) – Actual load lifted (W)**

$$\therefore W_f = W_i - W = (P \times \text{V.R.}) - W \qquad (\text{Since } W_i = P \times \text{V.R.})$$

Kinetic Energy

For an object with mass m and speed v, the **kinetic energy** is defined as

Kinetic energy is a scalar (it has magnitude but no direction); it is always a positive number; and it has SI units of kg · m^2/s^2 . This new combination of the basic SI units is known as the **joule**:

s^2

As we will see, the joule is also the unit of work W and potential energy U . Other energy units often seen are:

Work-When an object moves while a force is being exerted on it, then **work** is being done on the object by the force.If an object moves through a displacement **d** while a constant force **F** is acting on it, the force does an amount of work equal to

$$W = \mathbf{F} \cdot \mathbf{d} = F\, d \cos \varphi$$

where φ is the angle between **d** and **F**.Work is also a scalar and has units of 1 N · m. But we can see that this is the same as the joule

Work can be negative; this happens when the angle between force and displacement is larger than 90°. It can also be zero; this happens if $\varphi = 90°$. To do work, the force must have a component along (or opposite to) the direction of the motion.If several different (constant) forces act on a mass while it moves though a displacement **d**, then we can talk about the **net work** done by the forces,If the force which acts on the object is not constant while the object moves then we must perform an integral (a sum) to find the work done.Suppose the object moves along a straight line (say, along the x axis, from x_i to x_f) while a force whose x component is $F_x(x)$ acts on it. (That is, we know the force F_x as a function of x.) Then the work done isFinally, we can give the most general expression for the work done by a force. If an object moves from $\mathbf{r}_i = x_i\mathbf{i} + y_i\mathbf{j} + z_i\mathbf{k}$ to $\mathbf{r}_f = x_f\,\mathbf{i} + y_f\,\mathbf{j} + z_f\,\mathbf{k}$ while a force **F**(**r**) acts on it the workwhere the integrals are calculated along the path of the object's motion. This expression can be abbreviated asThis is rather abstract! But most of the problems where we need to calculate the work done by a force will just involve

Spring Force The most famous example of a force whose value depends on position is the **spring force**, which describes the force exerted on an object by the end of an **ideal spring**. An ideal spring will pull inward on the object attached to its end with a force proportional to the amount by which it is stretched; it will push outward on the object attached to its with a force proportional to amount by which it is compressed.If we describe the motion of the end of the spring with the coordinate x and put the origin of the x axis at the place where the spring exerts no force (the equilibrium position) then the spring force .Here k is **force constant**, a number which is different for each ideal spring and is a measure of its "stiffness". It has units of N/ m = kg/ s^2 . This equation is usually referred to as **Hooke's law**. It gives a decent description of the behavior of real springs, just as long as they can oscillate about their equilibrium positions and they are not stretched by too much!

The Work–Kinetic Energy Theorem One can show that as a particle moves from point $\mathbf{r}_i$ to $\mathbf{r}_f$, the change in kinetic energy of the object is equal to the net work done on it:

$$K = K_f - K_i = W_{net}$$

Power- In certain applications we are interested in the rate at which work is done by a force. If an amount of work W is done in a time t, then we say that the **average power** P due to the force

Conservative Forces

The work done on an object by the force of gravity does not depend on the path taken to get from one position to another. The same is true for the spring force. In both cases we just need to know the initial and final coordinates to be able to find W , the work done by that force.This situation also occurs with the general law for the force of gravity (Eq. 5.4.) as well as with the electrical force which we learn about in the second semester!This is a different situation from the friction forces studied in Chapter 5. Friction forces do work on moving masses, but to figure out how much work, we need to know how the mass got from one place to another.If the net work done by a force does not depend on the path taken between two points, we say that the force is a **conservative force**. For such forces it is also true that the net work done on a particle moving on around any closed path is zero.

Potential Energy For a conservative force it is possible to find a function of position called the **potential energy**, which we will write as U ($\mathbf{r}$), from which we can find the work done by the force.Suppose a particle moves from $\mathbf{r}_i$ to $\mathbf{r}_f$. Then the work done on the particle by a conservative force is related to the corresponding potential energy function by:

$$W_{ri \to rf} = - U = U(\mathbf{r}_i) - U(\mathbf{r}_f)$$

The potential energy U ($\mathbf{r}$) also has units of joules in the SI system.When our physics problems involve forces for which we can have a potential energy function, we usually think about the change in potential energy of the objects rather than the work done by these forces. However for non–conservative forces, we must directly calculate their work (or else deduce it from the data given in our problems).We have encountered two conservative forces so far in our study. The simplest is the force of gravity near the surface of the earth, namely $-mg\mathbf{j}$ for a mass m, where the y axis points upward. For this force we can show that the potential energy function is

$$U_{grav} = mgy$$

In using this equation, it is arbitrary where we put the origin of the y axis (i.e. what we call "zero height"). But once we make the choice for the origin we must stick with it.

The other conservative force is the spring force. A spring of force constant k which is extended from its equilibrium position by an amount x has a potential energy given by

Conservation of Mechanical Energy

If we separate the forces in the world into conservative and non-conservative forces, then the work–kinetic energy theorem says $W = W_{cons} + W_{non-cons} = \quad K$

But from Eq. 6.18, the work done by conservative forces can be written as a change in potential energy as:

$W_{cons} = - \quad U$

where U is the sum of all types of potential energy. With this replacement, we find:

$- \; U + W_{non-cons} = K$

Rearranging this gives the general theorem of the **Conservation of Mechanical Energy**:

$$K + \; U = W_{non-cons}$$

We define the **total energy** E of the system as the sum of the kinetic and potential energies of all the objects:

$$E=K+U$$

Then Eq. 6.21 can be written

$$E = \; K + \; U = W_{non-cons}$$

In words, this equation says that the total mechanical energy changes by the amount of work done by the non–conservative forces.

Many of our physics problems are about situations where all the forces acting on the moving objects are conservative; loosely speaking, this means that there is no friction, or else there is negligible friction.

If so, then the work done by non–conservative forces is zero, and Eq. 6.23 takes on a simpler form:

$$E = K + U = 0$$

We can write this equation as:

$$K_i + U_i = K_f + U_f \qquad \text{or} \qquad E_i = E_f$$

In other words, for those cases where we can ignore friction–type forces, if we add up all the kinds of energy for the particle's initial position, it is equal to the sum of all the kinds of energy for the particle's final position. In such a case, the amount of mechanical energy stays the same. . . it is conserved.

Energy conservation is useful in problems where we only need to know about positions or speeds but not time for the motion.

Work Done by Non–Conservative Forces

When the system does have friction forces then we must go back to Eq. 6.23. The change in total mechanical energy equals the work done by the non–conservative forces:

$$E = E_f - E_i = W_{non-cons}$$

(In the case of sliding friction with the rule $f_k = \mu_k N$ it is possible to compute the work done by the non–conservative force.)

Relationship Between Conservative Forces and Potential Energy (Optional?)

(the general expression for work W) and 6.18 give us a relation between the force **F** on a particle (as a function of position, **r**) and the change in potential energy as the particle moves from $\mathbf{r}_i$ to $\mathbf{r}_f$:

$$\int^{r_f} \mathbf{F} \cdot d\mathbf{r} = - U$$

Very loosely speaking, potential energy is the (negative) of the integral of **F**(**r**). Eq. 6.25 can be rewritten to show that (loosely speaking!) the force **F**(**r**) is the (minus) derivative of U (**r**). More precisely, the components of **F** can be gotten by taking partial derivatives of U with respect to the Cartesian coordinates:

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z} \qquad (6.26)$$

In case you haven't come across partial derivatives in your mathematics education yet: They come up when we have functions of several variables (like a function of x, y and z); if we are taking a partial derivative with respect to x, we treat y and z as constants.